Motor Planning Based on Sparse Command Representation

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1 Introduction: Recent computational studies have succeeded in presenting trajectory planning algorithms which could well replicate human movements. However, most of them have not explicitly clarified the problem of motor command representation. The authors[1] recently proposed a novel motor planning algorithm based on "sparse representation"[2], which tries to design motor commands with combination of a smaller number of basis patterns. This paper presents a generalized formulation of this framework.

2 Sparseness-Oriented Motor Planning: We assume that every motor command is represented as a linear combination of a fixed set of patterns (i.e., basis) $\{\phi_j(t)\}$, which is similar to the synergy decomposition model[3]. Precisely, a command pattern $u(t)$ is described as

$$u(t) = \sum_j a_j \phi_j(t - d_j), \tag{1}$$

where $\{a_j\}$ and $\{d_j\}$ are weights and time shifts, respectively. On this assumption, we determine the motor command for a given motor task by minimizing the following cost function,

$$\begin{aligned} &E\Big(\{a_j\}, \{d_j\}\Big|\text{Task}; \{\phi_j\}\Big) \\ &= [\text{Task Performance}] + \lambda[\text{Sparseness Preference}] \\ &= [\text{Endpoint Error}] + \lambda \sum_j |a_j|^p, \end{aligned} \tag{2}$$

where $p = 1$. Based on this criterion, the motor command to achieve the task with a sparse motor command is sought. Our previous study[1] showed that the optimum motor command under this criterion successfully generated appropriate commands for single-joint reaching movements.

3 Generalized Formulation: We generalize the above framework both in the command representation and optimization method. First, a command is represented by

$$u(t) = \sum_j a_j \phi_j(t), \quad \text{where } a_j \geq 0 \text{ and } \phi_j(t) \geq 0, \tag{3}$$

without the time-shift parameters, which simplifies the optimization procedure.

As for the optimization, we have to optimize "task performance" and "sparseness preference," simultaneously, which can be achieved either by maximizing task performance with sparseness preference or by maximizing sparseness under task requirement. For a reaching movement to a target θ_T, for example, the problem can be formulated either by

$$\text{min.} \sum_j |a_j|^p \ \text{ s.t. } \ |\theta(t) - \theta_T| \leq e_{max} \quad (t_f \leq t \leq t_f + t_p) \tag{4}$$

or by

$$\text{min.} \sum_{t=t_f}^{t_f+t_p} |\theta(t) - \theta_T|^2 \ \text{ s.t. } \ \sum_j |a_j|^p \leq a_{max}, \tag{5}$$

where $p = 1$ or 2, and t_f and t_p are movement time and target pausing time, respectively.

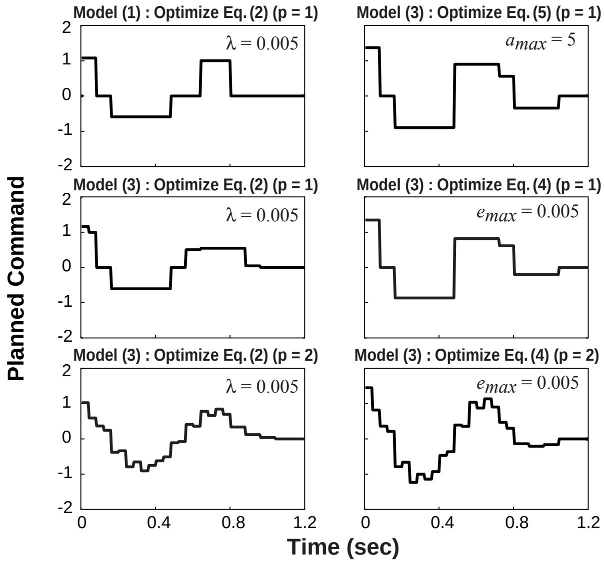

Fig. 1: Result of Experiment

Note that if the motor system is linear, the first problem can be solved using the linear and quadratic programming algorithms (LP and QP) when $p = 1$ and 2, respectively. Moreover, the second case resembles the "Lasso" model[4], which can be also solved using QP when $p = 1$. If the motor system is nonlinear, we optimize the cost function Eq.(2) to obtain the motor command.

4 Experiment: We ran a computer simulated experiment to examine whether or not above different formulations produced different command patterns.

Method: We adopted a single-joint reaching task as in ref.[1]. The pre-defined basis patterns were rectangular pulses with time-width of 40, 80, 160, 320 ms with the same L_2 norm. The endpoint error in Eq.(2) was evaluated by $|\theta(t) - \theta_T|^2$ during the pausing time t_p(=0.4 s), as in Eq.(5). The initial and target positions and movement time were fixed to 0 rad, 0.8 rad, and 0.8 s, respectively.

Result: Figure 1 summarizes the generated command patterns for six different formalizations. We can draw the following conclusions from this figure:

1. Different representation models (represented by Eqs. (1) and (3)) produced similar command patterns, though the command patterns depended on the value of λ.

2. Different optimization formulation (Eqs. (2), (4) and (5)) made little difference in the command patterns.

3. QP formulation tended to generate more continuous and minute patterns (i.e., containing more basis patterns) compared LP formulation, meaning that sparse representation can be obtained with L_1 constraints on the weights.

References

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